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## ELASTIC CALCULATIONS OF LIMITING MUD PRESSURES TO CONTROL HYDRO-FRACTURING DURING HDD

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**ABSTRACT:** Horizontal Directional Drilling has become an accepted trenchless construction technique for insertion of utility conduits and other buried pipe infrastructure. Drilling mud is used to stabilize the soil around the excavation zone, prior to pulling into place the new pipeline. Hydrofracturing is a related problem that is still not yet fully understood and whose consequences can be severe. It occurs when mud pressures within the excavated zone cause tensile fractures in the surrounding soil, and drilling mud flows through these fractures. This phenomenon is not only dependent on the drilling fluid pressure inside the newly created conduit, but the properties and stress state of the surrounding soil as well. The first step in a successful finite element model prediction of the soil response is to examine circumstances where the soil response is elastic. Finite element analyses were performed to model the response of undrained clays with varying soil properties. The elastic response of the soil was compared with a known closed-form solution to verify the accuracy of the model. The trends associated with varying the material properties of the host soil are discussed, and design formulae are presented to reflect soil and mud parameters, as well as the tensile strength of the soil. Results with another published design equation were examined, with that equation found to provide excessive estimates of frac-out pressures for cases when the soil does not experience shear failure.

### 1.0 INTRODUCTION

Horizontal Directional Drilling (HDD) is a trenchless construction technique that has been widely used in practice over the past decade. Since it is a trenchless technology, HDD allows buried conduits (such as sewer pipes) to be installed in areas with specific construction demands at a significantly reduced financial or environmental cost compared to traditional cut and cover trench construction. However there are still aspects of the practice that are not fully understood such as the uncontrolled fracturing of the soil surrounding the drilled conduit. This phenomenon, known as hydraulic fracturing or "frac-out", is affected by the pressures of the drilling fluid used to stabilize the excavated zone, but is not fully understood especially in relation to the surrounding ("host") soil. The consequences of hydraulic fracturing include reductions in drilling efficiency due to the loss of drilling fluid pressure, and ground heave causing substantial disruption of existing buried and surface infrastructure. New techniques have been developed to monitor drilling fluid (or "mud") pressures in the field (e.g., Baumert et al. 2004), but further work is needed to establish the mud pressures that lead to hydraulic fracturing.

The preservation of drilling fluid pressures in the borehole is important to maintain borehole stability during drilling and pulled-in-place pipe installation. Mud pressures less than 7.5% and greater than 125% of the overburden stress have been shown to allow plastic failure of the borehole (Duvestyn and Knight, 2000). To gain a better understanding of these pressure limits and the advent of hydraulic fracture, a 0.2m diameter conduit at various cover depths and mud pressures was analyzed using the finite element (FE) method. The analysis accounts for the coefficient of earth pressure at rest ( $K_0$ ), and quantifies the limits of elastic soil response as controlled by the undrained shear strength of the host soil.

The current state-of-the-practice includes use of an equation developed at the Delft University of Technology (Delft Geotechnics 1997), herein referred to as the Delft equation. This is defined for both frictional and cohesive soils, giving the maximum allowable down-hole drilling fluid pressure ( $P_{max}$ ). For cohesive soils, the undrained cohesion ( $c_u$ ) value is used along with a friction angle of zero and the equation can be reduced to:

$$P_{max} = \sigma_0 + c_u \quad [1]$$

where  $\sigma_0$  is the initial overburden stress. This equation does not account for the potential tensile strength of the soil. This is because reliable measurements of tensile strength are generally very difficult to obtain; the tensile strength is conservatively assumed to be zero for this and all other calculations presented here.

The primary purpose of this paper is to use elastic finite element analysis to illustrate the relationship between the drilling fluid pressures in a HDD installation and the onset of hydraulic fracturing (i.e., where there is no shear failure in the surrounding soil). A comparison is made with the Delft equation (Delft Geotechnics 1997).

## 2.0 REVIEW OF ELASTIC THEORY

The length of an HDD installation is much larger than the radius of the resulting borehole and therefore the problem of drilling the conduit can be simplified to two dimensional plane-strain conditions, This greatly simplifies the modeling processes, and permits a comparison with existing elastic theory for an infinite plate with a circular hole.

The shear and normal stresses in a plane of elastic material can be calculated relative to the known horizontal, vertical and shear stresses ( $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$ , respectively; Obert and Duval 1967). When these stresses are applied to the problem of an infinite plate with a circular hole and applied stresses in the x- and y-directions of  $\sigma_x$  and  $\sigma_y$ , respectively (Figure 1), Obert and Duval (1967) showed that the resulting tangential stress component becomes:

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) \left( 1 + \frac{a^2}{r^2} \right) - \frac{1}{2}(\sigma_x - \sigma_y) \left( 1 + \frac{3a^4}{r^4} \right) \cos 2\theta \quad [2]$$

where  $a$  = radius of the hole  
 $r$  = distance away from centre of the hole  
 $\theta$  = angle from the horizontal

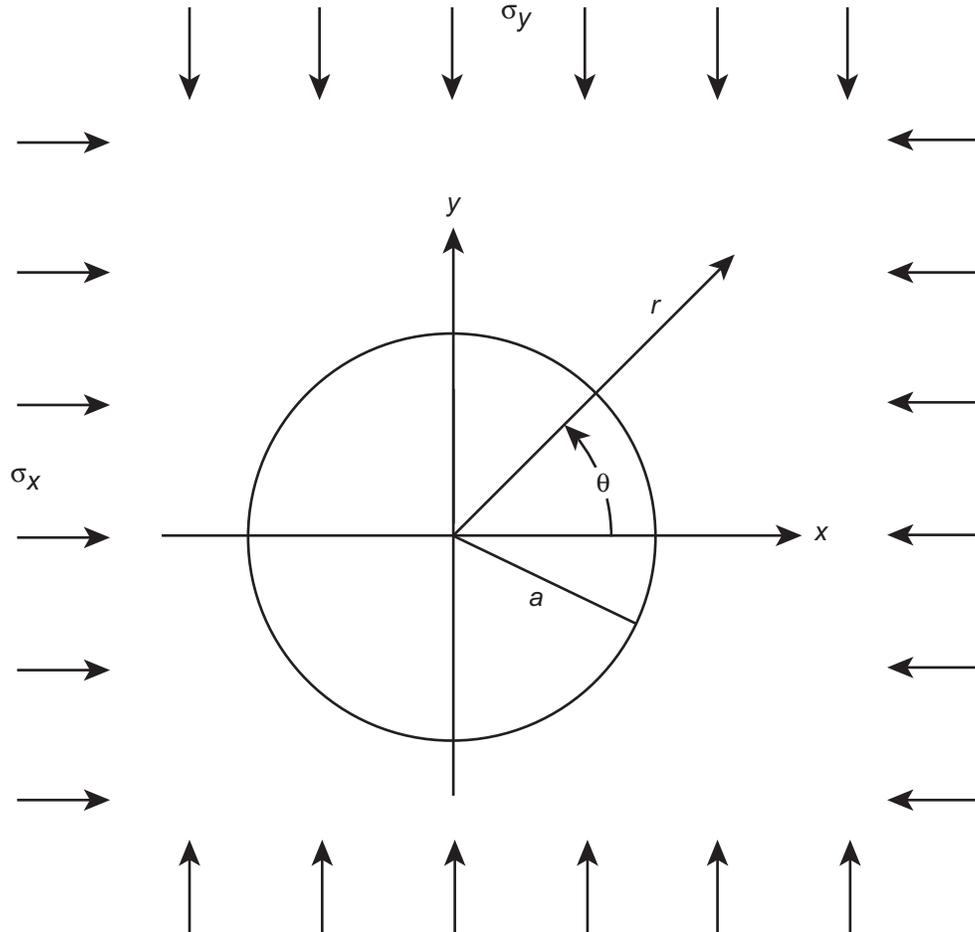
This and all other expressions given in this paper feature a compression positive sign convention.

Since the host soil has a  $K_0$  value of less than 1, the horizontal geostatic stress would be less than the vertical geostatic stress (i.e.  $\sigma_x$  would be less than  $\sigma_y$ , in this case). Therefore the critical tangential stress at the soil-hole boundary would occur at the crown or invert of the hole and can be calculated by simplifying Equation 2 where  $r = a$  and  $\theta = \pi/2$  to be:

$$\sigma_{\theta} = 3\sigma_x - \sigma_y \quad [3]$$

When the isotropic drilling fluid is added to the inside of the hole, the radial stresses are represented by the induced drilling fluid pressure ( $P_i$ ), also called the “mud pressure”. These radial stresses reduce the circumferential stress  $\sigma_{\theta}$  by  $-P_i$  in the soil immediately above the crown (Hefny and Lo 1992):

$$\sigma_{\theta} = 3\sigma_x - \sigma_y - P_i \quad [4]$$



**Figure 1** Circular hole in an infinite plate (modified from Obert and Duval 1967).

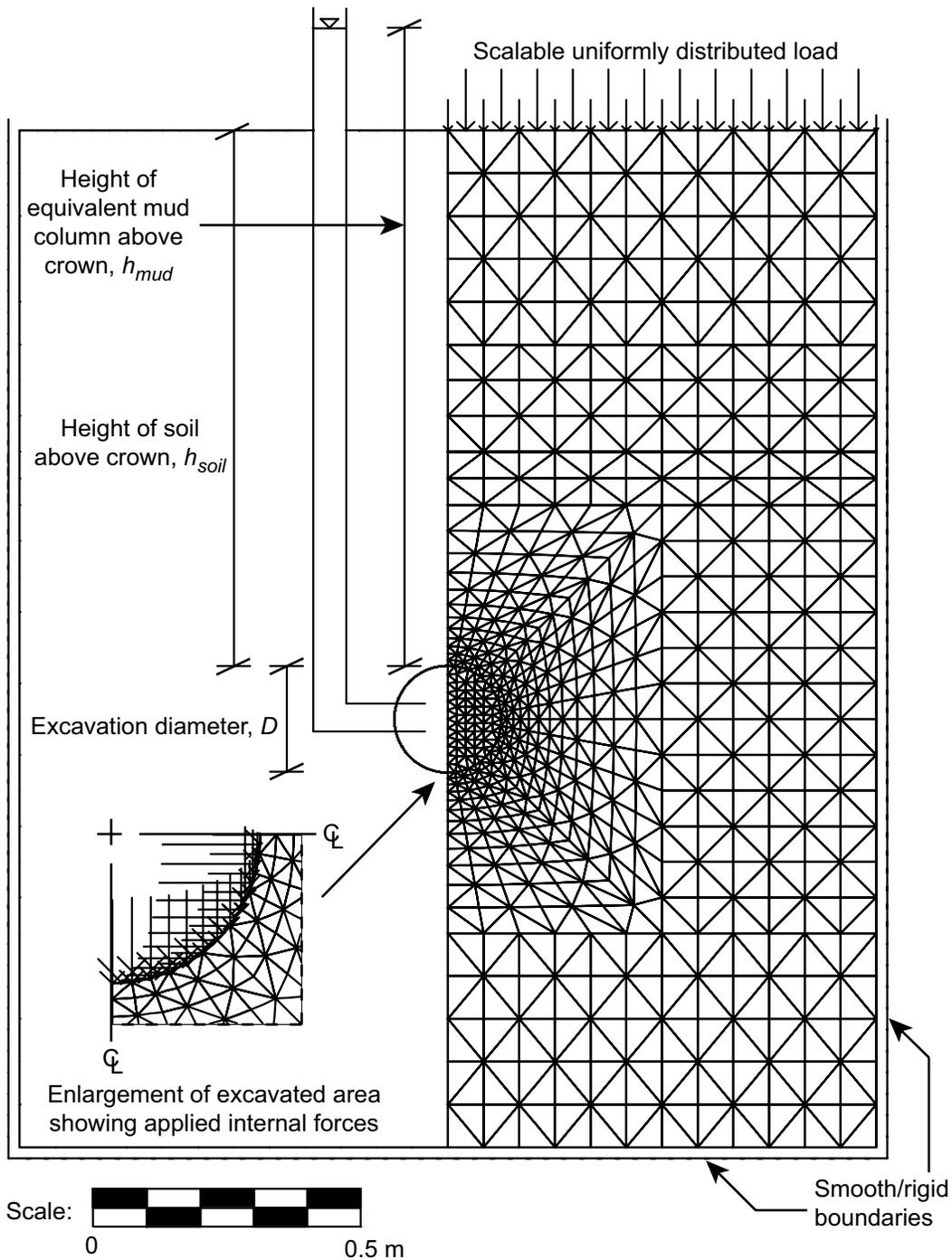
### 3.0 NUMERICAL MODEL

The simple plate theory represented by Equations 2 to 4 is based on a number of approximations:

1. the gradient in earth pressures across the cavity is neglected
2. the gradient of mud pressures across the cavity is neglected
3. the potential for shear failure in the soil around the cavity is not considered

The Delft equation is also based on certain approximations. Finite element analysis of the cavity drilling operation is able to examine the effect of these approximations on the tensile stresses that develop and potentially initiate hydraulic fracture. The finite element modeling was developed in a series of steps that were each examined in turn to ensure the effectiveness of that computer analysis.

The first step involves the development of a suitable finite element mesh to provide reliable calculations of stresses and displacements (Figure 2).



**Figure 2** Example mesh and parameter definition.

As shown, the mesh becomes finer near the circular area that was to be excavated in the model. The excavation was modeled under two-dimensional (plane-strain) conditions in order to simulate a cross section of the construction process. The soil was modeled using 2404 six-noded triangular elements with elasto-plastic behaviour and a Mohr-Coulomb failure criterion; however the first phase of the investigation reported here involved just the elastic response being studied and characterized. In order to reproduce the initial geostatic stresses within the soil block for an arbitrary depth of soil, a scaleable uniformly distributed load was applied to the top of the mesh to represent the soil above that position. This

technique is successful since the top of the mesh is located at a sufficient distance above the cavity being drilled so that the stiffness of the soil above the top of the mesh can be neglected without influencing the soil stresses around that cavity. The initial geostatic stresses were prescribed at the start of the analysis based on the unit weight of the soil ( $\gamma_{soil}$ ), and the coefficient of lateral earth pressure,  $K_0$ . The overall mesh dimensions were chosen to limit boundary effects (a total width and height of the mesh of eight and nine times the diameter of the hole, respectively).

Since the newly drilled hole in an HDD project is never an empty cavity, it was decided that the model would simultaneously simulate the incremental reduction of geostatic stresses within the conduit and the incremental addition of the drilling fluid pressures. This was performed by reducing the initial reaction forces of the soil to be excavated to zero at the same time as the final reaction forces corresponding to the drilling fluid pressures were increased to their full values from zero (Figure 2). In this manner, the model uses a simple stress path idealizing the combined effect of the soil removal and the application of mud pressures.

In order to generate the set of forces required to model the simultaneous reduction of geostatic stresses and the increase in drilling fluid pressure, the individual forces for each case were first estimated. The initial soil reactions ( $F_{soil}$ ) were calculated by finite element modeling of the initial geostatic conditions in the soil, and evaluating the normal reactions exerted around the annulus of the excavated area.

The final drilling fluid reactions were found in a similar manner, but employing the drilling fluid density and a fluid condition involving lateral pressures at each point equal to the vertical pressures. Those pressures are equivalent to the action of a column of drilling mud up to a representative depth,  $h_{mud}$ . The modeling of different drilling fluid pressures (or different 'mud depths') was achieved by separating the drilling fluid reaction forces into two sets. The first set  $F_f$  corresponds to the contributions from the linear distribution of the drilling fluid pressure across the excavated cavity (this is also equivalent to the weight of the drilling fluid in the cavity). The second set  $F_h$  is due to the action of the fluid pressure at the crown of the cavity. As the height of the drilling fluid column increases, the component associated with the linear distribution of mud pressures ( $F_f$ ) remained the same, while the component associated with the crown mud pressures ( $F_h$ ) increases.

#### 4.0 MODEL PARAMETERS

All calculations shown here were for a drilled cavity with a diameter ( $D$ ) of 0.2m, and a clayey host soil with a unit weight of  $16\text{kN/m}^3$  (Craig 1997). To investigate the effect of the existing geostatic stresses, the soil was modeled with  $K_0$  values of 0.6 and 0.9 (Craig 1997). These different  $K_0$  values change the final ground stresses by adjusting the magnitude of the initial horizontal stresses.

When adding the drilling fluid pressures during excavation, the soil was modeled as responding under undrained conditions, since the construction process is relatively quick compared to the hydraulic conductivity of the clayey host soil (the undrained and drained response of soil is discussed in detail by Craig, 1997). The unit weight of the drilling fluid ( $\gamma_{mud}$ ) was assumed to be  $13\text{kN/m}^3$  as per Andersen et al. (1994).

The undrained cohesion,  $c_u$ , was varied from 20kPa for a very soft clay, to 150kPa for a stiff clay according to values suggested in the Canadian Foundation Engineering Manual (Canadian Geotechnical Society 1992). Poisson's ratio for the soil during drilling and application of mud pressure was selected as 0.49, given the incompressible response expected for clay acting under undrained conditions, Craig (1997). The undrained elastic modulus,  $E_u$ , was selected based on its classification and undrained shear strength, using the correlation provided by the Electric Power Research Institute (1990); see Table 1. However, the uniform elastic modulus that is selected affects the ground deformations but has no real effect on the soil stresses resulting from the elastic analysis of the system. Analyses were undertaken with  $h_{soil}$  values of 2m (10 times the conduit diameter) and 5m (25 times the conduit diameter). The height of the drilling fluid column above the crown of the conduit was varied between zero and three times

the value of  $h_{soil}$ . In each analysis, the value of  $h_{mud}/h_{soil}$  was modeled in increments of 0.1 to effectively capture the trend of the soil response.

**Table 1** Soil properties for different clay materials.

Soil Type:	$c_u$ (kPa)	$E_u$ † (kPa)
Very Soft	20	1600
Soft	40	4000
Firm	80	8000
Stiff	150	18000

\* from Canadian Geotechnical Society 1992

† from Electric Power Research Institute 1990

## 5.0 FINITE ELEMENT ANALYSIS RESULTS

### 5.1 Model Evaluation and Boundary Effects

For verification purposes, the circular conduit was firstly modeled without any internal fluid pressures, permitting direct comparison with the elastic plate theory of Obert and Duval (1967). Three scenarios were examined to assess the model performance.

The simplest model included uniformly distributed loads on all sides of the mesh with no fixed boundaries, a  $K_0$  value equal to 1.0, and the unit weight of soil equal to zero to remove any gradients with depth (with zero gradients). The finite element results should then match those of elastic plate theory exactly. The calculated stresses at the crown and sides of the hole from the finite element solution did indeed compare well with the values obtained from elastic plate theory, with differences of only 0.3% and 0.1% between the computer and closed form calculations at the crown and sides of the cavity, respectively.

Next, the  $K_0$  value was reduced below 1.0 to examine performance for non-equal horizontal and vertical stresses. The unit weight of the soil was again maintained at zero to remove gradient effects. The difference between the finite element and elastic theory results at the crown and the sides of the hole then increased to 4.1% and 1.6%, respectively. These differences are higher because the finite element mesh does not have integration points along the vertical or horizontal axes of the cavity. Stresses calculated at integration points nearby will not then match plate theory at crown and sides. Given this limitation in the location of stress values, the results are still considered very acceptable.

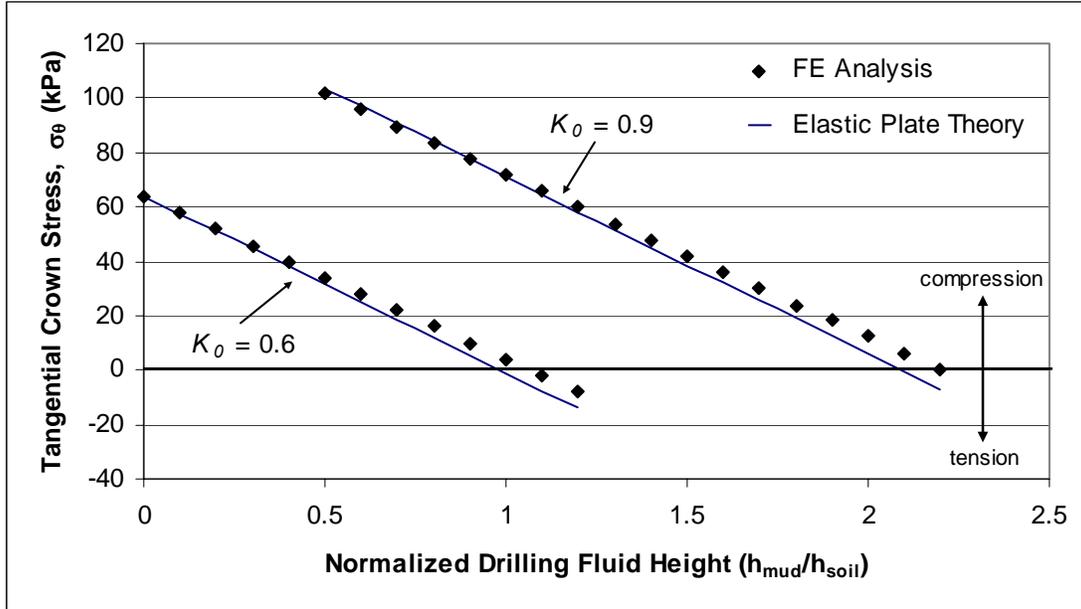
The final step added the soil unit weight, to simulate the gradients of geostatic stress with depth that occur in the field. The model maintained the  $K_0$  value below 1.0. Because the horizontal and vertical stresses in the model now vary with depth, the elastic plate theory results were calculated using the horizontal and vertical stresses that result at the depth of the level of the crown before the hole is opened in the soil. The difference between the finite element and elastic theory solutions at the crown and sides of the hole were again found to be within acceptable limits, rising slightly to 4.2% and 1.5%, respectively. These three comparisons illustrate that the addition of anisotropic initial stresses (vertical and horizontal stresses that are not equal) and gradients of earth pressures with depth have small effects on the outcome of the computer analysis.

### 5.2 Effect of Drilling Fluid Pressure

The drilling fluid height, and therefore the applied pressure inside the cavity, was varied to predict the response on the stresses in the surrounding soil. The resulting tangential crown stresses ( $\sigma_\theta$ ) were then compared to those arising from the elastic plate solution (Hefny and Lo 1992), Equation 4.

In all cases, the FE analysis followed the elastic plate solutions closely when no shear failure occurred in the surrounding soil. The internal fluid pressure applies a radial stress on the boundary of the hole ( $\sigma_r$ ) and directly affects the circumferential stress  $\sigma_\theta$  in the soil at the crown. As expected, as mud pressure  $\sigma_r$  increases, the circumferential stress  $\sigma_\theta$  decreases by an equal amount. This phenomenon is illustrated in Figure 3 which shows the change in tangential crown stress with increases in drilling fluid pressure for one cavity depth (5m) and two particular coefficients of lateral earth pressure (0.6 and 0.9). These results are typical of all those obtained for the suite of soil parameters and burial depths that were considered. Mud pressure is plotted in Figure 3 using the mud depth  $h_{mud}$  normalized by  $h_{soil}$ . In each case, the mud

pressure calculated using finite element analysis is slightly higher than the elastic plate calculations, so it should be conservative to use elastic plate theory to estimate the development of tensile fracture.



**Figure 3** Example plot of the tangential crown stress at different normalized mud depths for both elastic plate theory and finite element analyses (for the case of  $h_{soil} = 5\text{m}$ ,  $\gamma_{soil} = 16\text{kN/m}^3$ ,  $\gamma_{mud} = 13\text{kN/m}^3$ ).

When the tensile strength ( $\sigma_t$ ) of a given clayey soil is assumed to be zero, the maximum allowable normalized drilling fluid height can be read directly from a plot such as Figure 3. Assuming  $\sigma_t = 0$ , failure occurs when the tangential crown stress decreases to a value of zero. For  $K_0$  of 0.6, the maximum allowable normalized mud depth can be interpolated as 1.0. This means that for  $\gamma_{mud}$  of 13 kN/m<sup>3</sup>, the maximum allowable down-hole drilling fluid pressure is (1.0)(5m)(13 kN/m<sup>3</sup>) = 65 kPa. This value can also be calculated using an equation derived from the elastic theory used to obtain the two lines in Figure 3:

$$P_{\max} = \sigma_0(3K_0 - 1) \quad [5]$$

### 5.3 Effect of $K_0$

Since the response is effectively represented by the elastic plate theory (Obert and Duval 1967), a change in the coefficient of earth pressure at rest  $K_0$  simply shifts the line plotted in Figure 3, and this influences the maximum mud pressure as per Equation 5. This has a significant effect on the maximum allowable drilling fluid pressure, since a greater  $K_0$  value means a greater in-situ horizontal stress and therefore a larger initial tangential stress at the crown to be overcome by the pressure of the drilling fluid in the hole. For example, a  $K_0$  value of 0.9 results in a maximum down-hole drilling fluid pressure of 136 kPa. This is illustrated in Figure 3 which shows the significant difference between a soil with  $K_0 = 0.6$  and the same soil with  $K_0 = 0.9$ . The latter will feature a maximum drilling fluid height of more than twice that of the soil with the smaller  $K_0$  value.

### 5.4 Upper/Lower Bounds of Validity

Elastic plate theory (Obert and Duval 1967) using earth pressures at the level of the cavity crown effectively captures the response of the finite element analysis provided the soil remains in a fully elastic state. However, the finite element analysis no longer follows elastic plate theory once plastic yielding occurs in the host soil. The undrained cohesion of the soil can be used to establish the upper and lower bounds for mud pressures where the soil response can be effectively represented using elastic plate theory. The difference in major and minor principal stresses in the soil is limited to  $2c_u$ , so that the limits of drilling fluid pressures where the elastic plate theory is effective are given by:

$$P_{lower} = \frac{1}{2} \cdot \sigma_0 (3K_0 - 1) - c_u \quad [6]$$

$$P_{upper} = \frac{1}{2} \cdot \sigma_0 (3K_0 - 1) + c_u \quad [7]$$

### 5.5 Comparison with the Delft Equation

The Delft equation (Delft Geotechnics 1997) is seeing increasing use in design calculations of maximum mud pressures for HDD installations. It considers the cohesion of the soil as well as the friction angle, but does *not* include the coefficient of earth pressure at rest or the tensile strength of the soil. It does appear to account for the effect of shear failure in the soil. Comparison of maximum mud pressures obtained from the Delft equation and those obtained using elastic plate analysis presented in Equation 5 is used here to illustrate the effects of the different approximations inherent in the two different formulations.

Figures 4 and 5 compare the maximum allowable down-hole drilling fluid pressure ( $P_{max}$ ) calculated by the Delft equation (Equation 1, Delft Geotechnics 1997) and those obtained from Equation 5 over a range of  $K_0$  values from 0.3 to 1.5. Figure 4 starts with a comparison for the specific case of  $c_u=40\text{kPa}$ , whereas Figure 5 provides information pertaining to all of the clayey soil classes listed in Table 1 (from Very Soft to Stiff clays). These comparisons indicate that there are some considerable differences between the Delft and elastic plate solutions.

Firstly, Figure 4 shows that maximum mud pressures for  $c_u$  of 40kPa and  $K_0$  values from 0.3 to 0.67 are overestimated if the Delft Equation is employed. Provided  $K_0$  lies in this range, the soil response is actually elastic and the finite element analysis demonstrated earlier that Equation 5 provides a very effective estimate of the maximum mud pressure. The Delft equation is based on the assumption that initial earth pressures are isotropic ( $K_0=1$ ), and its formulation also appears to depend on the assumption that the ground is in a state of shear failure. For soil responding in the elastic range, the Delft equation appears neither valid nor conservative.

Shear failure at the crown develops in the 40kPa soil once  $K_0=0.67$ , and the elastic plate solution will not provide the correct mud pressure limit for  $K_0$  values higher than this. The Delft equation may provide the correct theoretical value of limiting mud pressure when  $K_0=1$ ; however, more work is needed to confirm that and to assess the mud pressures producing circumferential crown tension for elastic-plastic soil with anisotropic initial stress ( $K_0 \neq 1$ ).

Figure 5 presents calculations based on elastic plate theory, Equation 5, and Delft calculations for each of the four clay strength classes listed in Table 1. This figure reveals that for most  $K_0$  values the response of firm and stiff clays is elastic, and the mud pressures that lead to hydraulic fracturing in these soils depends on the coefficient of lateral earth pressure  $K_0$  rather than the shear strength (undrained cohesion) of the soil. Further work is needed to establish limiting mud pressures for  $K_0 > 0.5$  in very soft ( $c_u=20\text{kPa}$ ) soils, and for  $K_0 > 1.0$  in firm ( $c_u=80\text{kPa}$ ) soils.

## 6.0 SUMMARY AND CONCLUSIONS

While hydraulic fracturing of the soil above the crown of the cavity created during Horizontal Directional Drilling is often encountered, it is not well understood. Finite element analyses were performed to examine this phenomenon, to gain a better understanding of when and why it occurs. This analysis was used to examine the performance of elastic plate theory in calculating the mud pressures that lead to tensile fracture in the soil surrounding the cavity. A parametric study was conducted for an undrained clayey soil at construction depths of 2m and 5m, and with  $K_0$  values of 0.6 and 0.9, over a range of typical drilling mud pressures. The smallest tangential stress in the soil surrounding the newly drilled hole occurs at the crown, due to the effect of the geostatic stress increases with depth. Those tangential stresses in the soil at the crown were examined using both the finite element analysis and the elastic plate theory.

Provided the soil response is elastic, the maximum allowable down-hole drilling fluid pressure can be easily and accurately calculated using elastic plate theory, and an equation was introduced for calculation of the maximum drilling fluid pressure (the pressure that will initiate hydraulic fracture). Upper and lower

bound values of mud pressure were also derived to define the limits of elastic soil response as a function of clay shear strength. The so-called 'Delft' equation generally provides unconservative estimates of limiting mud pressure, likely because its formulation assumes that the initial earth pressures are isotropic (coefficient of lateral earth pressure is 1), and that the soil is in a state of shear failure. Further analysis of hydraulic fracture in elastic-plastic soils is needed to establish limiting pressures beyond the elastic range, and to provide more reliable design equations for use in guiding field operations.

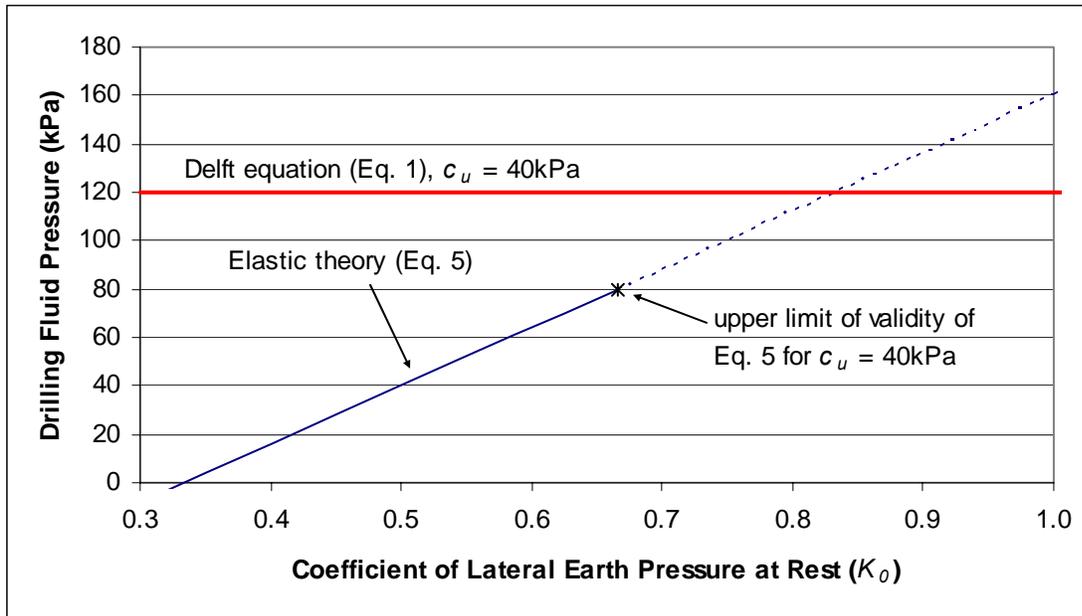


Figure 4 Maximum mud pressure versus  $K_0$ ; comparison with limiting pressure from Delft Geotechnics (1997);  $h_{soil} = 5\text{m}$ ,  $\gamma_{soil} = 16\text{kN/m}^3$ ,  $\gamma_{mud} = 13\text{kN/m}^3$ ,

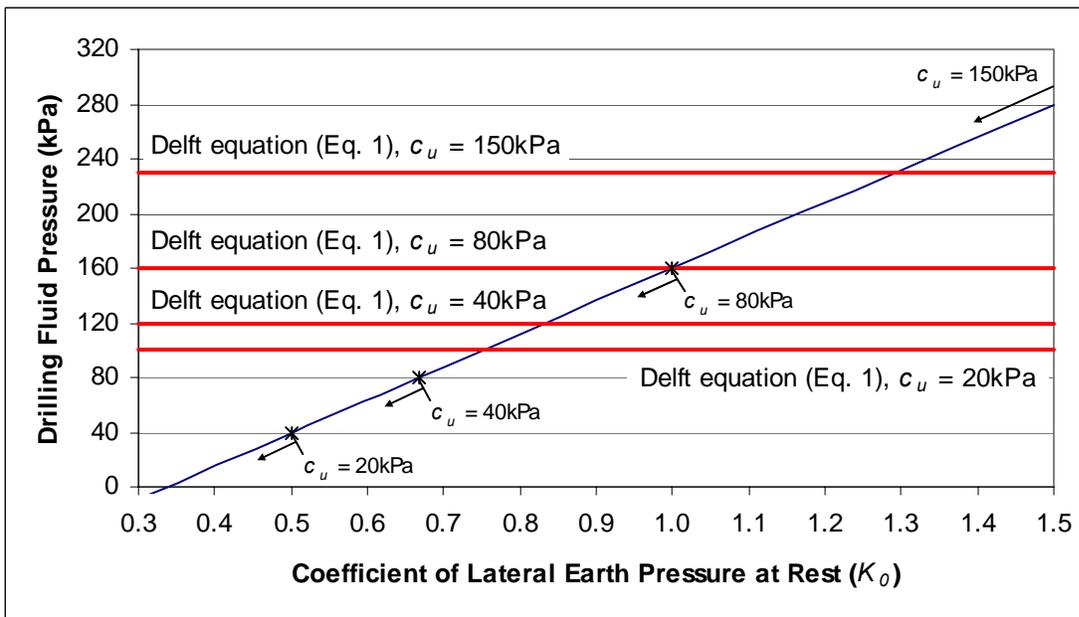


Figure 5 Maximum mud pressure versus  $K_0$ ; comparison with limiting pressure from Delft Geotechnics (1997);  $h_{soil} = 5\text{m}$ ,  $\gamma_{soil} = 16\text{kN/m}^3$ ,  $\gamma_{mud} = 13\text{kN/m}^3$ ,

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